



Counting and Permutations

Learning outcomes

- ▶ List outcomes of an experiment using:
 - Systematic listing
 - Two-way tables
 - Tree diagrams
- ▶ Apply fundamental principles of counting
- ▶ Count the arrangements of n distinct objects ($n!$)
- ▶ Count the number of ways of arranging r objects from n distinct objects

What to wear...



How many different outfits can be made?

6

How is my answer 6?

- ▶ Systematic listing
 1. Blue jeans and grey jumper
 2. Blue jeans and beige jumper
 3. Blue jeans and red jumper
 4. Black jeans and grey jumper
 5. Black jeans and beige jumper
 6. Black jeans and red jumper

6

How is my answer 6?

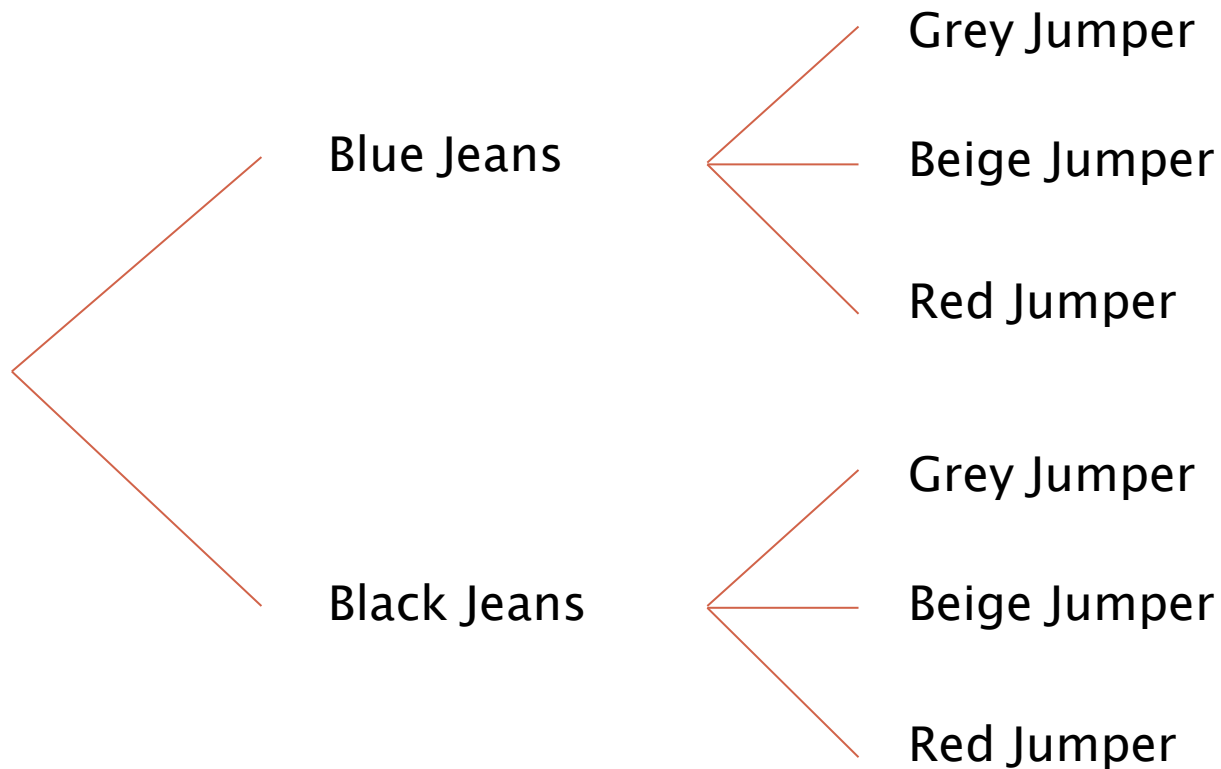
- ▶ Two-way table

	Grey jumper	Beige jumper	Red jumper
Blue Jeans	Blue, Grey	Blue, Beige	Blue, Red
Black Jeans	Black, Grey	Black, Beige	Black, Red

6

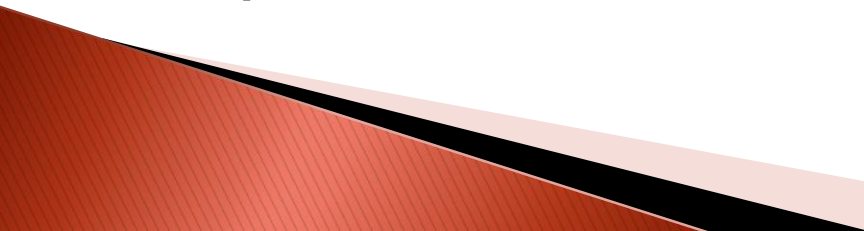
How is my answer 6?

▶ Tree Diagram



6

Terms

- ▶ **Sample space** is the listing of all possible outcomes
 - ▶ **Systematic listing** involves writing down all possible outcomes
 - ▶ **Two-way tables** are used to write down all possible outcomes when there are two sets of options
 - ▶ **Tree diagrams** are used to show all possible outcomes when there are two or more sets of options
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Fundamental principal of counting

- ▶ If one event has m possible outcomes and a second event has n possible outcomes,
- ▶ The total number of possible outcomes is:
 - $m \times n$
- ▶ In other words, to find the total number of outcomes:
 - multiply the number of possibilities together
 - Jeans \times Jumpers
 - $2 \times 3 = 6$

Exercise 1

1. How many different outfits can a girl choose from the following?



2. Draw a two way table to represent all possible outcomes

3. Draw a tree diagram to represent all possible outcomes

Exercise 2

1. How many different outfits can a girl choose from the following?



2. Represent all possible outcomes using an appropriate method

Further exercises

- ▶ A girl has 10 tops, three skirts and five pairs of shoes. In how many ways can she dress for the evening?
- ▶ In a restaurant there are four choices for starter, six choices for main course and 5 choices for desert. How many possible three course meals are there?
- ▶ A cinema has 10 films showing. Each film can be viewed in standard or 3D. How many choices of film are there?

Arrangements

- ▶ Sometimes we multiply a list of numbers in descending order. For example:
 - $5 \times 4 \times 3 \times 2 \times 1$
- ▶ This can be written as $5!$ Pronounced:
 - “Five factorial”
- ▶ Formula for $n!$:
 - $N(n-1)(n-2)(n-3)\dots$
 - $4 \times 3 \times 2 \times 1$

Example:

- ▶ In how many ways can the letters of the word “MONSOON” be arranged in each of the following:
 - If there are no restrictions

1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
7	6	5	4	3	2	1

- $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = 5,040$

Example:

- ▶ In how many ways can the letters of the word “MONSOON” be arranged in each of the following:
 - If the arrangement must begin with an “S”

1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
1	6	5	4	3	2	1

- $1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 1 \times 6! = 720$

Example:

- ▶ In how many ways can the letters of the word “MONSOON” be arranged in each of the following:
 - If the arrangement **must not begin with an “S”**
 - There are a total of 5,040 different arrangements
 - 720 different arrangements begin with an “S”
 - $5,040 - 720 = 4,320$

Example:

- ▶ In how many ways can the letters of the word “MONSOON” be arranged in each of the following:
 - If the arrangement must begin and end with a vowel

1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
3	5	4	3	2	1	2

- $3 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 = 3 \times 5! \times 2 = 720$

Permutations

${}^n P_r$ The Number of Ways of Arranging r Objects from n Distinct Objects

- ▶ In how many ways can the letters of the word “MONSOON” be arranged taking them 3 at a time?

1 st	2 nd	3 rd
7	6	5

- ▶ $7 \times 6 \times 5 = 210$

Permutations

${}^n P_r$ The Number of Ways of Arranging r Objects from n Distinct Objects

$${}^n P_r = \frac{n!}{(n-r)!}$$

- ▶ n is the total number of objects to choose from
- ▶ r is the total number of objects we have to arrange in order
- ▶ So like the word monsoon:

$$\frac{7!}{(7-3)!} \longrightarrow \frac{7!}{4!} \longrightarrow 210$$

Exercises

- ▶ An iPhone has 7 songs on it. 5 songs are played in sequence. How many possible sequences are there?
- ▶ There are 40 items of clothing in your wardrobe. You must choose 2 at a time. How many possible selections are there?

Re-cap

- ▶ We learnt how to:
 - Calculate the number of outcomes when given choices. Eg: 2 pants, 3 jumpers = 6 possible outcomes
 - Sample space:
 - Systematic listing
 - Two-way table
 - Tree diagram
 - Arrangements Eg: 5!
 - Permutations: The Number of Ways of Arranging r Objects from n Distinct Objects

$${}^n P_r$$