Powers and roots

This presentation was created as a help math material for the students of a first year of the graduation branches. This part of math can be quite difficult, students often have not necessary imagination.

Parts of this presentation are also used in lessons of non-graduation students of the technical branches. These students have less interest of these problems but it is very useful for them when they know how to solve them.

POWERS WITH NATURAL EXPONENT

- Power with a natural exponent is every term in a form a^n
- a is a real number
- n is a natural number
- We can also write the above mentioned as:

$a \cdot a \cdot \ldots \cdot a$, where the number of a is n:

$$a \cdot a \cdot \ldots \cdot a = a^n.$$

exercise

- Please, solve:

2^6

$(-3)^4$

$(-3)^3$

-3^4

$\left(\frac{2}{3}\right)^2$

$0,2^5$

$(-0,3)^3$

64

81

-27

-81

$\frac{4}{9}$

$\frac{32}{100\,000} = 0{,}000\,32$

$\frac{-27}{1000} = -0{,}027$

The sentences about a calculating with the powers

For every $a,b \in R$, for every $r,s \in N$

$a^r . a^s = a^{r+s}$ — **Multiplication of the powers**

$a^r : a^s = \frac{a^r}{a^s} = a^{r-s}, a \neq 0, r > s$ — **Division of the powers**

$(a^r)^s = a^{r.s}$ — **Power of the power**

$(a.b)^r = a^r . b^r$ — **Power of the multiplication**

$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}, b \neq 0$ — **Power of the division**

exercise – rearrange and solve

a) $10^2.10^4$

b) $(-3)^2.(-3)$

c) $(-5)^{64}:(-5)^{61}$

d) $(2^3)^3$

e) $2^5.5^5$

f) $\left(\frac{-2}{5}\right)^3$

g) $\left(\frac{4}{3}\right).\left(\frac{3}{4}\right)^3$

a) 1 000 000

b) - 27

c) - 125

d) 512

e) 100 000

f) result → $\frac{-8}{125}$

g) result → $\frac{9}{16}$

THE POWERS WITH A NEGATIVE EXPONENTS

- The power with a negative exponent is every term a^n
- a is a real number, n is a whole number

For solving we can use the above mentioned sentences

and more…

For every a,b ≠ 0 real and every m ∈ Z

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m} = \left(\frac{1}{a}\right)^m$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

exercise – rearrange and solve

10^{-4}

$(-3)^{-4}$

$\left(\frac{5}{2}\right)^{-3}$

$0{,}04^{-2}$

$2^{-3} + 3^{-2}$

$\left(\frac{2}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-3}$

$\frac{1}{10\,000}$

$\frac{1}{81}$

$\frac{8}{125}$

625

$\frac{17}{72}$

$\frac{265}{4} = 66{,}25$

THE POWER WITH A RATIONAL EXPONENT

- The power with a rational exponent is every term a^n
- a is a real number, n is a rational number

For solving we can use the above mentioned sentences

and more …

$$\sqrt[s]{a^r} = a^{\frac{r}{s}}, \qquad s \in N,\ r \in Z,\ a > 0$$

exercise – rearrange and solve

$a^{\frac{2}{3}}.a^{\frac{3}{4}}$ → $a^{\frac{17}{12}}$

$b^{0,3}.b^{-3,3}$ → $b^{-3}=\frac{1}{b^3}$

$\frac{c^{\frac{1}{2}}.c^{\frac{1}{3}}}{c}$ → $c^{\frac{-1}{6}}$

$\left(a^{\frac{2}{3}}.b^{\frac{4}{3}}\right)^{\frac{3}{2}}$ → $a.b^2$